Multiple Quadratic Snake Software for Medical Image Segmentation

Yoichi Nakaguro†
Stanislav Makhanov†
†Sirindhorn International Institute of Technology
Thammasat University, Thailand
{ynakaguro,makhanov}@siit.tu.ac.th

Abstract

Classical active contour models provide an elegant framework for optimal estimation in image processing. However, their adaptability to image noise and convergence speed have been major challenges. In this paper, we introduce an image segmentation software based on a multiple quadratic multiple snake model coupled with the GVF external force. We describe the detailed implementation of the software and explain an effective strategy for choosing correct input parameters.

Keywords: Active Contours, Quadratic Snakes, Snake Multiplicity, GVF, Adaptive Initialization

1 Introduction

Flexible automated systems capable of extracting structures and regions of interest from digital images encompass a broad range of applications. For example, applications such as automated road extraction and object detection in medical images would boost the productivity of technicians enormously. Manual marking and extraction of those objects is an extremely slow and laborious process considering the degree of complexity that those objects can possess in the real world. Towards the ultimate goal of fully automated image object extraction, there has been a great deal of progress in the computer vision and image processing communities. But after 30 years of research, (see [1] for a thorough review), there is still no system attaining the speed, robustness, and level of automation necessary for practical application on arbitrary imagery.

Classical active contour models [2; 3; 4] provide an elegant framework for optimal estimation in image processing; rather than writing an algorithm to extract the object or region of interest, we simply consider an energy functional whose minimum of which is achieved at a good solution. Then, given a new image, we use general optimization techniques to find a contour minimizing the energy functional.

Although the classical models should fundamentally work well, in practice, their adaptability to image noise and convergence speed become major challenges. For noise, image enhancement based on object types such as oriented filtering applied to road extraction applications, can greatly improve extraction results [5]. For convergence speed, Xu and Prince’s gradient vector flow (GVF) [6] technique makes it possible to achieve fast convergence of snakes initialized far from the boundary of the object. GVF also improves convergence to objects with concave boundaries where the classical snake models fail.

Rochery et al. have proposed a geometric model for higher-order active contours, in particular quadratic snakes, for extraction of linear structures like roads [7]. The idea is to use a quadratic formulation of the contour’s geometric energy to encourage anti-parallel tangents on opposite sides of a road and parallel tangents along the same side of a road. These priors increase the final contour’s robustness to partial occlusions and decrease the likelihood of false detections in regions not shaped like roads.

In this paper, we introduce an image segmentation software based on a multiple quadratic multiple snake model coupled with the GVF external force. We describe the detailed implementation of the software and explain an effective strategy for choosing correct input parameters. In our preliminary experiments using ultrasound images with tumors, our software successfully segmented the tumors with practical time cost (a few seconds).

2 Methods

2.1 Quadratic snake model

This section briefly overviews our quadratic snake model by extending the original snake model [2] with quadratic geometric energy [7].

We define a closed 2D spline \( \tilde{\gamma}(p) = \)
that minimizes energy functional

\[ E(\gamma) = \int E_g(\gamma, \gamma_p, \gamma_{pp}, p) dp = E_g + E_i, \quad (1) \]

where \( \gamma_p \) and \( \gamma_{pp} \) are first and second partial derivatives of a contour \( \gamma \), respectively.

\[ E_g(\gamma) = \int \left( \frac{1}{2} \alpha \gamma_p^2 + \frac{1}{2} \beta \gamma_{pp}^2 \right) dp \]

\[ -\frac{\delta}{2} \int \overline{u}(p_1) \cdot \overline{u}(p_2) \Psi(\|\gamma(p_1) - \gamma(p_2)\|) \ dp_1 \ dp_2, \quad (2) \]

The first integral represents the stretch energy and bending energy, respectively. \( \alpha \) and \( \beta \) are the weights of those energy terms. A large stretch energy encourages the contour to be shortened, on the other hand, a large bending energy forces the contour to be straightened. \( \overline{u}(p) \) is the unit-length tangent to \( \gamma \) at point \( \gamma(p) \), and \( \Psi(z) \), given the distance \( z \) between \( p_1 \) and \( p_2 \), is used to weight the interaction between those two points. For positive \( \delta \), \( E_g(\gamma) \) is minimized by contours with short length and parallel tangents.

The quadratic term is responsible for interactions between points on the snake. The sigmoid function \( \Psi(\cdot) \) defines the radius of the region in which anti-parallel tangents should be discouraged:

\[ \Psi(z) = \begin{cases} 
1 & \text{if } z < d - \epsilon, \\
0 & \text{if } z > d + \epsilon, \\
\frac{1}{2} \left( 1 - \frac{z - d}{\epsilon} \right) - \frac{1}{2} \sin \frac{\pi}{2} \frac{z - d}{\epsilon} & \text{otherwise.} 
\end{cases} \quad (3) \]

\( d \) is approximate width of the region of interest and \( \epsilon \) is the sharpness of the sigmoid function.

\( E_i \) is the image energy that depends on the image intensity \( I(x, y) \) as follows

\[ E_i(\gamma, I) = -\int \lambda |\nabla I|^2 dp, \quad (4) \]

where \( \lambda \) is a weight.

Contour \( \gamma \) is a solution of the following Euler equation:

\[ -\frac{d^2}{dp^2} \left( \frac{\partial E_g}{\partial \gamma_{pp}} \right) + \frac{d}{dp} \left( \frac{\partial E_g}{\partial \gamma_p} \right) - \frac{\partial E_i}{\partial \gamma} = 0. \quad (5) \]

\[ \text{2.2 GVF external force} \]

To improve the convergence, we adopt Xu and Prince’s gradient vector field (GVF) [6] technique. The GVF is a vector field \( \overrightarrow{V}(x, y) = [u(x, y) \ v(x, y)]^T \) that minimizes an energy functional given by

\[ E(\overrightarrow{V}) = \int \int \mu (u_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla I|^2 |\nabla V - \nabla I|^2 \ dx \ dy, \quad (6) \]

where

\[ u_x = \frac{\partial u}{\partial x}, \ u_y = \frac{\partial u}{\partial y}, \ v_x = \frac{\partial v}{\partial x}, \ v_y = \frac{\partial v}{\partial y}. \]

Minimizing the functional leads to an Euler equation given by

\[ \mu \nabla^2 \overrightarrow{V} - (\overrightarrow{V} - \nabla I) |\nabla I| = 0. \quad (7) \]

The Laplacian creates a slow varying field in homogeneous regions causing elimination (diffusion) of noise and false boundaries. With large \( |\nabla I| \), the energy is dominated by the second term. Therefore the energy achieves the minimum, when \( \overrightarrow{V} = \nabla I \), encouraging the snake to snap to the object boundary.

\[ \text{2.3 Snake multiplicity} \]

A single quadratic snake [7] is unable to extract enclosed regions and multiple disconnected characters in an image. We address this limitation by introducing a family of cooperating snakes that are able to split, generate offspring (anti-snakes), disappear, and merge if necessary.

In our formulation, specifying the points on \( \gamma \) in a counterclockwise direction creates a shrinking snake, whereas specifying the points on \( \gamma \) in a clockwise direction creates a growing snake.

\[ \text{2.3.1 Splitting a snake} \]

We split a snake into two snakes whenever two of its arms are squeezed too close together.

\[ \text{2.3.2 Deleting a snake} \]

A snake \( \gamma \) is deleted if it has perimeter less than \( L_{\text{delete}} \).

\[ \text{2.3.3 Merging two snakes} \]

We merge two growing snakes when they come too close to each other.
2.3.4 Generating an anti-snake

The evolution of the snake continues until it either attaches itself to a boundary of the character or gets deleted. In the first case an anti-snake is generated by offsetting the original snake by $d_{\text{offset}}$ pixels inside the object.

2.4 Adaptive initialization

With the ability of anti-snake generation, it is possible that we simply initialize a single shrinking snake along the boundary of an image and extract all inner objects. For medical images with complex shape objects, however, more sophisticated initialization is generally required to obtain correct convergence. For this purpose, we employ an adaptive initialization approach in which we initialize multiple shrinking/growing snakes depending on the intensity of image regions.

3 Multiple Snake Software

3.1 Software implementation

The software is a C program with four classes: Snake, SnakeFamily, Evolution, and ExternalForces (see Figure 1). The Snake class defines the fundamental representation of a snake as dynamic array data structure. Also, utility methods to obtain contour related data such as length, area, and local curvature are also defined. The SnakeFamily class defines unit operations related to the multi-snake framework (splitting, deleting, merging, and duplicating). The Evolution class contains an implementation of Euler equation solver using the gradient descent technique. Finally, the ExternalForces class defines an interface to manage external vector forces such as image gradient and the GVF force.

3.2 Input parameters

All input parameters related to snake evolution are configurable in the left pane of the software interface (see Figure 2). For segmentation of objects in medical images, we recommend the following choice of input parameters:

- The weight in the snake functional (4) $\lambda = 1.2$,
- The weight in Equation (2) $\alpha = 40.0$,
- The weight of the quadratic term in Equation (2) $\beta = 16.5$,
- The parameters of the sigmoid function (3) $d = 10.5, \epsilon = 0.45$,
- The weight parameter in the GVF functional (6) $\mu = 0.2$,
- The splitting parameters (see section 2.3.1), $d_{\text{split}} = 1.2, \eta = 0.5$,
- The deleting parameter (see section 2.3.2), $L_{\text{delete}} = 20$,
- The offset to generate the anti-snake (see section 2.3.4) $d_{\text{offset}} = 1.8$.

It should be noted that the model can be sensitive to variations of $\lambda$. Therefore, this parameter is evaluated using the “brutal force” approach. The parameter producing the best average segmentation accuracy should be selected. As opposed to that, $\alpha$ and $\beta$ which control the snake evolution admit a relatively large range, which is practically independent on the training set, namely, $5 \leq \alpha \leq 100, 0.5 \leq \beta \leq 30$.

Furthermore, the parameters of the sigmoid function (3) $d$ and $\epsilon$ are designed to define the size of the region in which anti-parallel tangents should be discouraged. Therefore, roughly $d$ is the average size of the object. If the snake points are closer than $d$, they repel each other, otherwise they do not communicate. $\epsilon$ is the size of the transition region from $\Psi = 1$ to $\Psi = 0$. We recommend $0.1 \leq \epsilon \leq 0.7$. The weighting (regularization) parameter for the GVF model has correlation with the noise. The higher the image noise the larger $\mu$ should be (see for instance [8]). The regularization parameter can be selected visually. The anti-parallel vectors attached to the boundary of the object and the absence of the star-like artifacts corresponding to the noise indicate an appropriate $\mu$.

The step $\eta$ controls the numerical discretization of the contour. We recommend a sub-pixel accuracy $0 < \eta < 1.0$. For instance, $\eta = 0.7$ worked well for many experiments. It is also our experience that too small $\eta$ may lead to a slow convergence. Furthermore, parameter $d_{\text{split}}$ does not depend on the size of the objects. However, to guarantee continuity of the contours after the split, $d_{\text{split}}$ should be less than $2\eta$. Therefore,
in our case $0 < d^{\text{split}} < 1.4$. In order to prevent unnecessary multiple splits, the length of the two newborn snakes $L_1$ and $L_2$ should satisfy $L_1 > L^{\text{split}}$ and $L_2 > L^{\text{split}}$, where $L^{\text{split}}$ is the smallest allowed length of the new snake. We recommend $L^{\text{split}} = 10$. Note that $L^{\text{delete}} = 20$, therefore, if $L_1$ or $L_2$ is less than $L^{\text{delete}}$, the new snake gets deleted immediately.

4 Conclusion

In this paper, we introduce an image segmentation software based on a multiple quadratic multiple snake model coupled with the GVF external force. We describe the detailed implementation of the software and explain an effective strategy for choosing correct input parameters. In our preliminary experiments using ultrasound images with tumors, our software successfully segmented the tumors with practical time cost (a few seconds). For the system to be more practical, however, we should automate the initialization procedure in future.

References


Figure 2. Software interface.

