Dynamic State Estimation for Vehicle Platoon System Based on Feedback Estimators

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Abstract

A dynamic feedback system is developed for estimating the headway and velocity in a longitudinal three-vehicle platoon. The estimation system is modeled using a particle filter (PF) and an unscented Kalman filter (UKF) that estimate them by measuring the acceleration rate and/or velocity of probe vehicle(s) in the platoon. State equations are defined as a discrete conservation equation of headway and velocity, whereas the measurement equation is based on a conventional car-following model. The UKF and PF have the advantage of avoiding first-order approximation when implementing a filtering process to increase the estimation accuracy. Numerical analyses using real car-following data showed that the PF and UKF reduce the estimation errors in most cases at satisfactorily levels around 1m for headway and 1m/s for velocity estimations.

Keywords: Vehicle Platoon, State Estimation, Particle Filter, Unscented Kalman Filter, Feedback Estimation

1 Introduction

A vehicle platoon is a system that consists of multiple car-following vehicles moving longitudinally on an arterial road or freeway corridor. In order to achieve a high level of safety of vehicle platoon, not only the collision avoidance system of a single vehicle but also the cooperative control of multiple vehicles are required. Some studies have been proposed to evaluate and mitigate the risk of rear-end collision over a three-vehicle platoon by designing an appropriate human machine interface (HMI) [1, 2]. However, applying the HMI to a large platoon is still a difficult task because the sensing data are not obtained and shared in real-time by all platoon vehicles in the present car-following situation. Unfortunately, all the vehicles are not always equipped with a data collection system that can measure the headway and velocity. To bring a risk mitigation system to the real world in near future, an intelligent system is required that estimate the headway and velocity of all platoon vehicles indirectly from the measurement variables of some part of prose vehicles out of the platoon.

The authors have been attempting to develop a system of estimating the headway and velocity of a three-vehicle platoon using extended Kalman filter (EKF), a Neural Kalman Filter (NKF), Particle Filter (PF) and an Unscented Kalman Filter (UKF) [3, 4]. The study by Suzuki and Nakatsuji [5] showed that PF and UKF which require no first-order approximation in the model development performed the best among the other estimators such as EKF and NKF. Especially, PF is well known as a powerful tool and has been widely applied to various dynamic estimations such as traffic states estimation [5], estimating visual shape and motion [6], mobile robot control [7], target tracking [8] and so on.

As demonstrated above, PF performs very well in various non-linear estimation problems including the identification of headway and velocity of vehicle platoon. However, our previous study of vehicle platoon estimation confirmed the performance of PF in the simulated artificial car-following data only. Applicability of PF still remains unknown for real car-following situations.

This research aims to evaluate whether the PF and UKF estimate the velocity and headway of a three-vehicle platoon accurately even in real car-following scenarios. Also, performance of these two estimators is evaluated and compared by changing the number of measurement variables. Here, in this paper, only PF and UKF are selected as the feedback estimators since it has been proved that they performed the best compared to the existing approaches such as EKF and NKF.
2 Theoretical Background

2.1 State space model

The following state and measurement equations are defined as the state-space model, which is required for feedback estimation:

\[ x_k = f(x_{k-1}) + v_{k-1}, \]  
\[ y_k = g(x_k) + w_k, \]

where

- \( x_k \): state variables at time \( k \),
- \( y_k \): measurement variables,
- \( v_k \): system error,
- \( w_k \): measurement error,
- \( f \), \( g \): possible non-linear functions.

Let \( y_{1:k} \) and \( x_{1:k} \) denote as a set of all available measurement and state variables up to time \( k \) given by:

\[ y_{1:k} \equiv \{ y_1, y_2, \ldots, y_k \}, \]

\( x_{1:k} \equiv \{ x_1, x_2, \ldots, x_k \} \).

The estimation problem is to calculate the posterior probability \( p(x_k|y_{1:k}) \) when giving a set of all measurements \( y_{1:k} \). If measurement \( y_k \) is available, the posterior is updated through the Baye’s rule:

\[ p(x_k|y_{1:k}) = \frac{p(x_k|y_{1:k-1}) \cdot p(y_k|x_k)}{p(y_k|y_{1:k-1})}. \]

Here, \( p(y_k|x_k) \) is a likelihood function which describes the likelihood of \( x_k \) when giving the measurement \( y_k \). \( p(x_k|y_{1:k-1}) \) is a prior probability of \( x_k \) given by:

\[ p(x_k|y_{1:k-1}) = \int p(x_k|x_{k-1}) \cdot p(x_{k-1}|y_{1:k-1}) dx_{k-1}. \]

Also, \( p(y_k|y_{1:k}) \) is defined as:

\[ p(y_k|y_{1:k-1}) = \int p(y_k|x_k) \cdot p(x_k|y_{1:k-1}) dx_k. \]

There is a strong restriction that the posterior and prior probability density functions (PDF) \( p(x_k|y_{1:k}) \) and \( p(x_k|y_{1:k-1}) \) should be Gaussian in a conventional Kalman filter, whereas the Particle Filter (PF) does not require any assumptions or analytical functions for the PDFs.

2.2 Particle filter (PF) [9, 10, 11]

The key idea of particle filter (PF) is to represent the required posterior PDF by a set of random samples with associated weights and to compute the estimates based on these samples and weights [10].

Let \( \{ x_{1:i-1}, w_{1:i-1} \}^M_{i=1} \) denote as a set of particles and its associated weights. It is assumed that \( \{ x_{1:i-1}, w_{1:i-1} \}^M_{i=1} \) is given from the posterior PDF at time step \( k-1 \). When giving the measurement \( y_k \) at time step \( k \), the PF is to compute and update the particles and weights \( \{ x_{1:k}, w_{1:k} \}^M_{i=1} \) to \( \{ x_{1:i-1}, w_{1:i-1} \}^M_{i=1} \).

The weight is updated through the appropriate process called Sequential Importance Sampling (SIS). The SIS generates the particles based on the proposal distribution \( q(x) \) which is different from the objective distribution \( p(x) \) and then gives each particle the appropriate weight so as to make the particles to be close to the objective PDF.

The update process of weight is given by:

\[ \tilde{W}_{k}^{(i)} = \frac{w_{k}^{(i)} \cdot p\left(x_{k}^{(i)} \mid x_{k-1}^{(i)} \right) \cdot p\left(y_{k} \mid x_{k}^{(i)} \right)}{q\left(x_{k}^{(i)} \mid x_{k-1}^{(i)}, y_{1:k} \right)} \quad (i = 1, 2, \ldots, M). \]

The PF is the feedback estimator that theoretically places the random particles in the probability field to yield the accurate posterior PDF based on the Baye’s theory. It is guaranteed that the estimates through the filtering process are suboptimal. The estimation process of PF is as follows:

1. Initial sampling: Generate the initial particles \( \tilde{x}_{0}^{(i)} (i = 1, 2, \ldots, M) \) at \( k = 0 \) from the initial
probability density function \( p(\hat{x}_0) \).

2. Importance sampling: Generate one-step prediction of samples \( \hat{x}_k^{(i)} \) and \( \hat{y}_k^{(i)} \) based on the state and measurement equations:

\[
\hat{x}_k^{(i)} = f(\hat{x}_{k-1}^{(i)}) + v_{k-1}^{(i)},
\]

\[
\hat{y}_k^{(i)} = g(\hat{x}_k^{(i)}) + w_k.
\]

3. Importance weight: Calculate the importance weight \( \tilde{w}_k^{(i)} \) for each sample by the following equation:

\[
\tilde{w}_k^{(i)} = \frac{P(\hat{x}_k^{(i)}|\hat{y}_{k-1}^{(i)})P(y_k|\hat{x}_k^{(i)})}{q(\hat{x}_k^{(i)}|\hat{y}_{k-1}^{(i)}, y_{1:k})}.
\]

4. Resampling: The particle \( \hat{x}_k^{(i)} \) is updated into \( \hat{x}_0^{(i)} \) in proportion to the importance weight \( \tilde{w}_k^{(i)} \).

5. Normalize weight: Normalize the importance weight to \( \tilde{w}_k^{(i)} \) by

\[
\tilde{w}_k^{(i)} = \frac{\tilde{w}_k^{(i)}}{\sum_{j=1}^{M} \tilde{w}_k^{(j)}}.
\]

6. Estimation: From the updated particles \( \hat{x}_0^{(i)} \), compute the estimate by

\[
\hat{x}_k \approx \frac{1}{M} \sum_{i=1}^{M} \hat{x}_0^{(i)} = \frac{1}{M} \sum_{i=1}^{M} \tilde{w}_k^{(i)} \hat{x}_k^{(i)}.
\]

2.3 Unscented Kalman filter (UKF)

The UKF is another of the family of derivative-free Kalman filters, which require no calculation of partial derivative of state and measurement equations [12]. The conventional extended Kalman filter (EKF) provides the first-order approximation to the optimal estimates, whereas the UKF captures the posterior mean and covariance accurately to their second-order Taylor expansions using a minimal set of carefully chosen sample points called sigma points [12].

The first step to apply the UKF is to generate the sigma point as:

\[
\sigma_{i,p} = (\sqrt{N + \lambda} P_{x})_i,
\]

\[
\hat{\Phi}_{i,k-1} = \hat{x}_{k-1} + \sigma_{i,p} \quad (i = 1, \ldots, N),
\]

\[
\hat{\Phi}_{i,k-1} = \hat{x}_{k-1} - \sigma_{i,N-p} \quad (i = N + 1, \ldots, 2N),
\]

\[
\hat{y}_k = f(\hat{\Phi}_{k-1}),
\]

where \( \lambda \) is the scaling parameter.

One-step prediction of state variables and the error covariance are then given by:

\[
\hat{x}_k = \sum_{i=0}^{2N} h_i \hat{\Phi}_{i,k},
\]

\[
M_k^x = \sum_{i=0}^{2N} h_i \left[ \hat{\Phi}_{i,k} - \hat{x}_k \right]\left[ \hat{\Phi}_{i,k} - \hat{x}_k \right]^T + V_{k-1},
\]

where \( h_i \) is the weight defined by

\[
h_i = \frac{1}{2(N + \lambda)}.
\]

The same procedure is applied to compute \( \hat{y}_k, M_k^y \), and \( M_k^{xy} \) by defining the sigma vectors \( \hat{\Psi}_k \) and \( \hat{\Omega}_k \) as follows:

\[
\sigma_{i,M} = \sqrt{(N + \lambda) M} \hat{x}_k,
\]

\[
\hat{\Omega}_{i,k} = \hat{x}_0,
\]

\[
\hat{\Omega}_{i,k} = \hat{x}_k + \sigma_{i,M} \quad (i = 1, \ldots, N),
\]

\[
\hat{\Omega}_{i,k} = \hat{x}_k - \sigma_{i,M} \quad (i = N + 1, \ldots, 2N),
\]

\[
\hat{\Psi}_k = g(\hat{\Omega}_k),
\]

\[
\hat{\Psi}_k = \sum_{i=0}^{2N} h_i \hat{\Psi}_k,
\]

\[
M_k^{y} = \sum_{i=0}^{2N} h_i \left[ \hat{\Psi}_{i,k} - \hat{y}_k \right]\left[ \hat{\Psi}_{i,k} - \hat{y}_k \right]^T + W_{k-1},
\]

\[
M_k^{xy} = \sum_{i=0}^{2N} h_i \left[ \hat{\Phi}_{i,k} - \hat{x}_k \right]\left[ \hat{\Psi}_{i,k} - \hat{y}_k \right]^T.
\]

The Kalman gain is computed by (30) from the error covariance \( M_k^{xy} \) and \( M_k^y \). Also, the optimal estimate \( \hat{x}_k \) and \( P_k \) are updated through (31) and (32).
\[ K_k = M_k^{T} \left( M_k^{yy} \right)^{-1}, \quad (30) \]
\[ \dot{x}_k = \ddot{x}_k + K_k (y_k - \ddot{y}_k). \quad (31) \]
\[ P_k = M_k^{xx} - K_k M_k^{xx} K_k^T. \quad (32) \]

3. State Space Model for Vehicle Platoon

Assuming that three vehicles form a longitudinal platoon system, as depicted in Figure 1, the dynamics of headway and velocity of each platooned vehicle are defined as
\[ \ell_i^k = \ell_i^{k-1} + \left[ v_i^{k-1} - v_i^{k-1} \right] \Delta t, \quad (33) \]
and
\[ v_i^k = v_i^{k-1} + a_i^{k-1} \Delta t, \quad (34) \]
where
- \( \ell_i^k \): headway of vehicle \( i \) at time \( k \),
- \( v_i^k \): velocity,
- \( a_i^k \): acceleration rate,
- \( \Delta t \): discrete time step.

![Figure 1. Three-vehicle platoon](image)

The acceleration rate in Equation (34) is given by a conventional car-following model, which is well known as the Gazis-Herman-Rotery (GHR) model [13]:
\[ a_i^{k+T} = \alpha \left( \frac{v_i^k}{\ell_i^k} \right)^{m} \left[ v_i^{k-1} - v_i^{k} \right]. \quad (35) \]
where
- \( \alpha, m, n \): model parameters,
- \( T \): vehicle reaction time.

For simplicity, the reaction time is assumed to be zero, i.e. \( T = 0 \), under the assumption that errors due neglecting \( T \) will be minimized by the feedback process of the estimator. For the same reason, it is assumed that \( \alpha = m = n = 1 \). Substituting Equation (35) into (34) redefines the velocity as
\[ v_i^k = v_i^{k-1} + \alpha \left( \frac{v_i^{k-1}}{\ell_i^{k-1}} \right)^{m} \left[ v_i^{k-1} - v_i^{k-1} \right] \Delta t. \quad (36) \]

Equations (33) and (36) are regarded as the state equations, whereas Equation (35) can be used as the measurement equation for the state space model of a vehicle platoon. If velocity is also chosen as a measurement variable, the additional measurement equation is:
\[ v_i^k = v_i^k. \quad (37) \]

When assuming that the 3rd vehicle is the only probe car, i.e. the only car equipped with a sensing system, the estimation problem is reduced to precisely estimating the state variables such as headway and velocity of the 2nd and 3rd vehicles by observing the measurement variables of acceleration and/or velocity of the 3rd vehicle. When observing acceleration only, the state and measurement variables are:
\[ x_k = [\ell_2, v_2, \ell_3, v_3]^{T}, \quad y_k = [v_3]^{T}, \quad (38) \]
otherwise,
\[ x_k = [\ell_2, v_2, \ell_3, v_3]^{T}, \quad y_k = [v_3, a_3]^{T}. \quad (39) \]

It is preferable that the velocity of the 1st vehicle is also added to the measurement variables. However, \( v_1^k \) is not included in the measurement variables in this analysis, but explicitly given to the platoon system since the transition of \( v_1^k \) is not described and modeled in the state equation.

4. Numerical Analyses

4.1. Preparing Real Car-Following Data Sets

Real car-following data including headway \( \ell_i^k \), velocity \( v_i^k \) and acceleration rate \( a_i^k \) of three vehicles were collected in a field test using a test truck of the Japan Automobile Research Institute. In the test, a three-vehicle platoon travelling at a steady speed of around 60 km/h was made to decelerate and come to a complete stop, and this process was repeated. The deceleration rate was random from 1 to 5 m/s².

Thirteen scenarios (S1 to S13) which are suitable for the analysis were selected and used for the evaluation.
4.2 Evaluation Result for Measuring Acceleration Only

Figures 2 and 3 compare the headway and velocity estimates of scenario S2 among two estimators, PF, UKF and no-filter case. PF and UKF yield more accurate estimates than no-filter case for all state variables. Especially in the headway estimation of 2nd and 3rd vehicles, PF and UKF reduced the unexpected under estimation that was seen in no-filter case. Both PF and UKF are very close to the observed target, but UKF is more stable and accurate than PF. UKF seems to absorb the system and measurement noises to yield stable estimates.

Figures 4 and 5 depict the mean and standard deviation of root mean square errors (RMSE) of four state variables among thirteen scenarios. The t-test showed that mean errors of both headway estimates are statistically smaller than the no-filter case at 1% confidence level. The absolute error around 1 m, which is equivalent to the error by laser radar, is also acceptable as the satisfactorily level.

In the velocity estimations, however, no statistical difference is observed except the PF in the velocity estimates of 2nd vehicle although PF and UKF seem to decrease the mean errors. But, the absolute mean error around 0.8 to 1.2 m/s can be considered as the acceptable level for the collision risk evaluation.

Measuring the acceleration rate of 3rd vehicle is enough to yield accurate estimates of both headway and velocity. This is because the platoon size is very small and the three vehicles are traveling in an appropriate car-following situation without any unexpected events.

4.3 Evaluation Results for Measuring both Acceleration and Velocity

The Estimates by PF and UKF when measuring both acceleration and velocity are depicted in Figures 6 and 7 for the same scenario S2. This scenario is a nominal case that additional measurement variable increases the estimation accuracy. Unfortunately, however, there are some scenarios where the estimates are slightly worse by adding the velocity of 3rd vehicle as a measurement variable.

Figures 8 and 9 evaluate the estimation errors in average. The mean errors still remain small at the satisfactorily levels for both headway and velocity estimations. When comparing Figures 4 and 8 or 5 and 9, however, there are some cases where the error is increased by adding the velocity of 3rd vehicle.
vehicle as a measurement variable. The reason why the accuracy is worse when adding the measurement variable is not identified yet. The appropriate measurement variables that give us optimum estimates are depending on the platoon size or the position of probe cars. We need more sophisticated investigation to find out them by increasing the platoon size or changing the location of probe cars. If limited in this analysis, however, measuring the acceleration rate of the 3rd vehicle is enough to estimate the headway and velocity accurately.

Figure 4. RMSE of headway estimates when observing acceleration only (upper: 2nd vehicle, lower: 3rd vehicle)

Figure 5. RMSE of velocity estimates when observing acceleration only (upper: 2nd vehicle, lower: 3rd vehicle)

Figure 6. Headway estimates when observing acceleration and velocity of 3rd vehicle (scenario S2; upper: 2nd vehicle, lower: 3rd vehicle)

Figure 7. Velocity estimates when observing acceleration and velocity of 3rd vehicle (scenario S2)
Concluding Remarks

This research attempts to develop an intelligent feedback system for estimating dynamic states of three-vehicle platoon. Particle filter (PF) and unscented Kalman filter (UKF) are applied to estimate the headway and velocity indirectly from the acceleration rate and/or velocity of a prove vehicle. Numerical analyses showed that even when using real car-following data, the estimation accuracy is as low as a satisfactorily level compared to the case where no filtering process is applied. Also, it was found that only the acceleration rate of prove car is enough to provide an appropriate estimation precision for the headway and velocity estimations. However, this finding is limited only in the smallest platoon which consists of solely three vehicles.

More number of measurement variables or prove cars may be required when the platoon size is increased. Further work should be carried out to apply the estimation to a larger platoon system with longer time period to evaluate whether the proposed algorithm by PF or UKF is applicable to the real world car-followings.

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